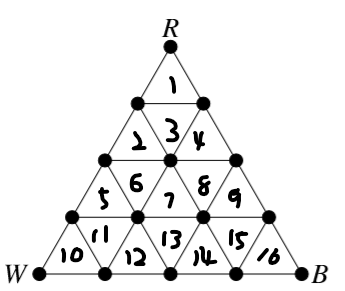
Haorui Chen 10407315 36111-cwk1-F-Formulating Arguments

1. For every edge in a finite graph, it is defined on 2 vertices, which means that adding an edge in a finite graph contributes to increase of total degree of a graph by 2. Imagine we have a finite graph G={V,E}, where E={∅}, hence total degree of the graph is 0. If adding an edge each time will increase the total degree by 2, when we have n edges in a graph G then its total degree is 2n, a even number. If there exists a graph in which there exist an odd number of vertices that with odd degree, since no matter how many vertices with even degree there are in the graph, total degree of the graph will be an odd number which contradicts with the common rule held by all finite graphs. Therefore, every finite graph has an even number of vertices with odd degree.
2. Consider a complete graph Kd+1. It is the most difficult graph among all graphs with a maximum degree of d in graph coloring problem, since it has the most constraints. If Kd+1 is (d+1)-colorable, then all the rest graphs are (d+1)-colorable. In such a graph, there are (d+1) vertices. If we are using d or less different colors for the graph, then obviously coloring cannot be achieved as there are always a pair of vertices that share the same color. However, if we are using (d+1) different colors, we can assign each vertex a different color, thus (d+1) coloring is achieved. Therefore, Kd+1 admits (d+1) coloring; and any other rest graphs that the maximum degree of any vertex is d admits (d+1) coloring.
3. a) Degree of X is at least 1, at most 4.

b)



Mark these triangles as the graph above. The first row gives all possible assignment of colors to vertices, the first column represents all small triangles in the graph. For each triangle there are some color assignments that are not applicable, so correspondent cell is left blank.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | R, R, R | W,W,W | B,B,B | R,R,W | R,R,B | W,W,R | W,W,B | B,B,W | B,B,R | W,R,B |
| 1 | 0 |  |  | 0 | 0 |  |  |  |  | 1 |
| 2 | 0 | 0 |  | 0 | 0 | 0 | 2 |  |  | 1 |
| 3 | 0 |  |  | 0 | 0 | 0 | 2 | 2 | 0 | 1 |
| 4 | 0 |  | 0 | 0 | 0 |  |  | 2 | 0 | 1 |
| 5 | 0 | 0 |  | 0 | 0 | 0 | 2 |  |  | 1 |
| 6 | 0 | 0 |  | 0 | 0 | 0 | 2 | 2 | 0 | 1 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 1 |
| 8 | 0 |  | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 1 |
| 9 | 0 |  | 0 | 0 | 0 |  |  | 2 | 0 | 1 |
| 10 |  | 0 |  |  |  | 0 | 2 |  |  | 1 |
| 11 |  | 0 |  | 0 | 0 | 0 | 2 | 2 | 0 | 1 |
| 12 |  | 0 | 0 |  |  | 0 | 2 | 2 | 0 | 1 |
| 13 |  | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 1 |
| 14 |  | 0 | 0 |  |  | 0 | 2 | 2 | 0 | 1 |
| 15 |  | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 1 |
| 16 |  |  | 0 |  |  |  |  | 2 | 0 | 1 |

c ) to violate the rule in the question, we need to show that every small triangle can be colored only by at most 2 colors. For any triangle at the corner, we can assign(say) color 1 and 2 t it, then all triangles that share a common edge with that triangle have to assign rest of their vertices with color 1 or 2, considering constraints on edges. When we do this to all small triangles, we get the whole graph colored only in 2 colors, which contradicts with the offset in question. Therefore, there is at least one small triangle which has vertices colored red, white and blue.